

Low Energy Chiral Lagrangian Parameters for Scalar and Pseudoscalar Mesons

W. Barddeen^a, E. Eichten^{*a} and H. Thacker^b

^aFermilab, PO Box 500, Batavia, IL 60510

^bDept. of Physics, University of Virginia, Charlottesville, VA 22901

We present results of a high-statistics study of scalar and pseudoscalar meson propagators in quenched QCD at two values of lattice spacing, $\beta = 5.7$ and 5.9 , with clover-improved Wilson fermions. The study of the chiral limit is facilitated by the pole-shifting ansatz of the modified quenched approximation. Pseudoscalar masses and decay constants are determined as a function of quark mass and quenched chiral log effects are estimated. A study of the flavor singlet η' hairpin diagram yields a precise determination of the η' mass insertion. The corresponding value of the quenched chiral log parameter δ is compared with the observed QCL effects. Removal of QCL effects from the scalar propagator allows a determination of the mass of the lowest lying isovector scalar $q\bar{q}$ meson.

1. Quenched Chiral Perturbation Theory

The lowest order chiral Lagrangian supplemented by the chiral symmetry breaking terms L_5 of $O(p^2 m_\pi^2)$ and L_8 of $O(m_\pi^4)$ (which model the mass-dependence of the chiral slope for the meson decay constants) is given by [1]:

$$\begin{aligned} \mathcal{L} = & \frac{f^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f^2}{4} \text{Tr}(\chi^\dagger U + U^\dagger \chi) \\ & + L_5 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U (\chi^\dagger U + U^\dagger \chi)) \\ & + L_8 \text{Tr}(\chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi) + \mathcal{L}_{\text{hairpin}} \end{aligned} \quad (1)$$

where

$$\mathcal{L}_{\text{hairpin}} \equiv -\frac{1}{2} m_0^2 \frac{f^2}{8} (i \text{Tr} \ln(U^\dagger) - i \text{Tr} \ln(U))^2 \quad (2)$$

We use explicit formulas for the pseudoscalar masses, and pseudoscalar and axial vector decay constants, consistent through order p^4 (for details see [1]). The coefficients L_5, L_8 follow the notation of Gasser and Leutwyler [2].

2. Pseudoscalar masses and chiral parameters

The expression for the pseudoscalar mass squared up to first order in the hairpin mass, L_5 and L_8 is:

$$\begin{aligned} M_{ij}^2 = & \chi_{ij}(1 + \delta I_{ij}) \{1 + \frac{1}{f^2} 8(2L_8 - L_5)\chi_{ij}[1 \\ & + \delta(\tilde{I}_{ij} + I_{ij})] + \frac{1}{f^2} 8L_5\chi_{ij}\delta\tilde{J}_{ij}\} \end{aligned} \quad (3)$$

where r_0 is a slope parameter, $\chi_i = 2r_0 m_i$, $\chi_{ij} = (\chi_i + \chi_j)/2$ and $m_i \equiv \ln(1 + 1/(2\kappa_i) - 1/(2\kappa_c))$. Here $\tilde{I}_{ij} = (I_{ii}\chi_i + I_{jj}\chi_j)/\chi_{ij}$ and $\tilde{J}_{ij} = J_{ij}/\chi_{ij}$. The loop integrals I_{ij} and J_{ij} are defined in [1].

For the pseudoscalar decay constants:

$$\begin{aligned} f_{P,ij} = & \sqrt{2}f r_0 (1 + 0.25\delta(I_{ii} + I_{jj} + 2I_{ij})) \{1 \\ & + \frac{4}{f^2}(4L_8 - L_5)\chi_{ij}[1 + \delta(\tilde{I}_{ij} + I_{ij})] \\ & - \frac{4}{f^2}L_5\delta\chi_{ij}[(\frac{\tilde{I}_{ij}}{2} + I_{ij}) - (\tilde{J}_{ii} + \tilde{J}_{jj})]\} \end{aligned} \quad (4)$$

While for the axial vector decay constants:

$$\begin{aligned} f_{A,ij} = & \sqrt{2}f(1 + 0.25\delta(I_{ii} + I_{jj} - 2I_{ij})) \\ & \{1 + \frac{4}{f^2}\chi_{ij}L_5[1 + \delta(2I_{ij} - \tilde{I}_{ij})]\} \end{aligned} \quad (5)$$

We study 350 configurations on a $16^3 \times 32$ lattice with $\beta = 5.9$. Clover fermions ($C_{SW} = 1.50$) and the MQA procedure [3] was used. We consider six κ values (.1397, .1394, .1391, .1388, .1385, .1382) with $\kappa_c = .140143$. The fits are shown in Figures 1-2.

*Presenter

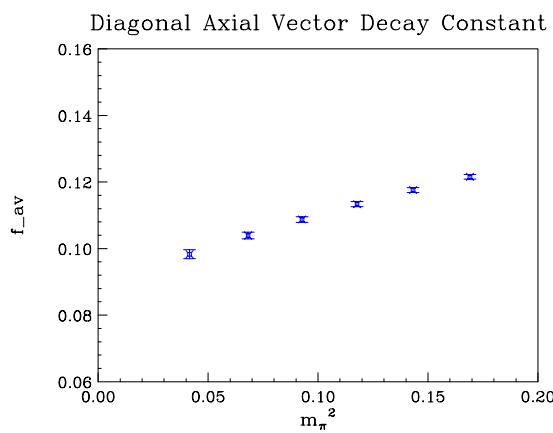
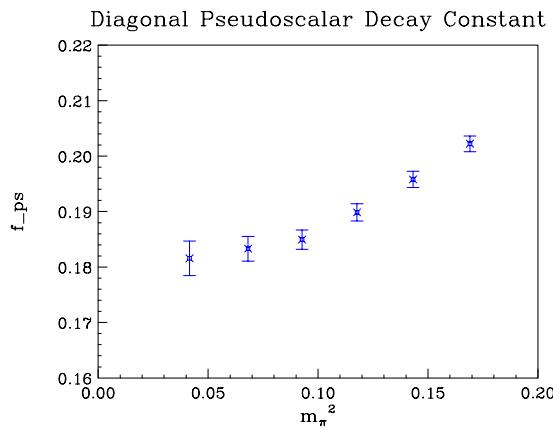
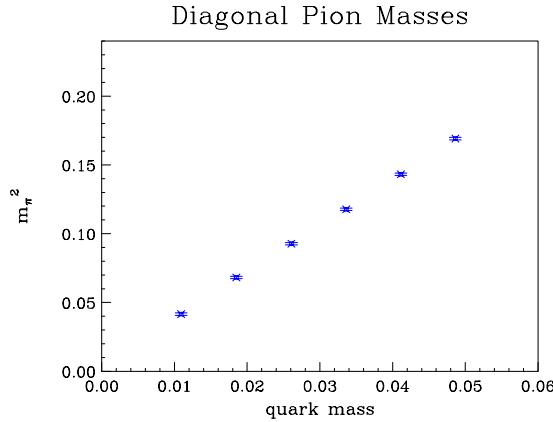


Figure 1. m_π^2 , f_{ps} and f_{av} versus diagonal masses.

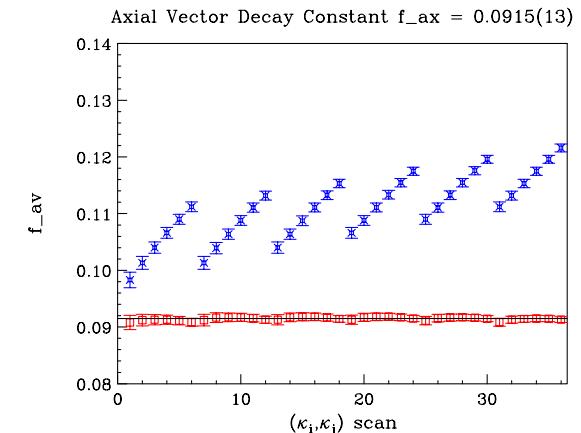
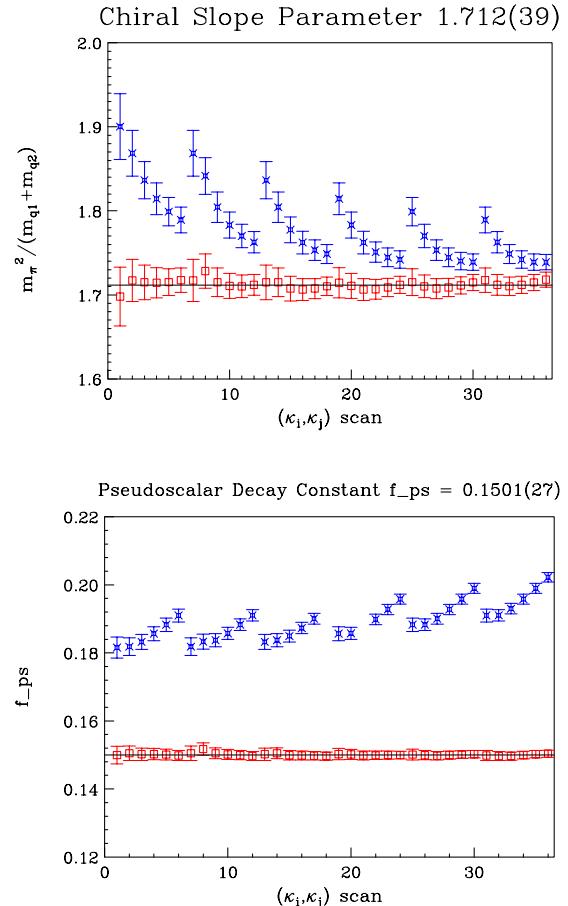


Figure 2. m_π^2 , f_{ps} and f_{av} and ratios to chiral fits.

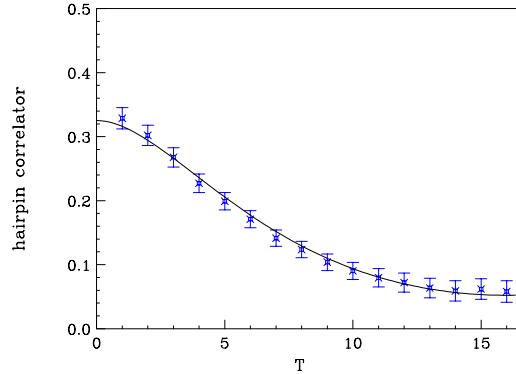


Figure 3. Hairpin propagator and double pole fit for $\kappa = .1394$.

3. Hairpins and Scalar Propagators

In the quenched theory the disconnected part of the eta prime correlator (hairpin term) has a double pole $\frac{-m_0^2}{(p^2+m_\pi^2)^2}$ whose coefficient determines m_0 . The correlator for $\kappa = .1394$ and a double pole fit ($m_\pi = 0.261$ and $m_0 = 0.211$) is shown in Figure 3. The δ parameter is given by:

$$\delta = \frac{m_0^2}{16\pi^2 f_\pi^2 N_f} \quad (6)$$

As the quenched chiral limit is approached the isovector scalar correlator shows negative norm behaviour. The correlators for the lightest and heaviest κ values are shown in Figure 4.

Properly accounting for the effects of the hairpin - pion bubble[4] allows a good fit of the isovector scalar correlator for all quark masses. One output is the (a_0) mass.

4. Results

Using the MQA technique, meson properties (masses and decay constants) can be extracted with sufficient accuracy to allow a fit of higher order chiral parameters, L_5 and L_8 . The physical results for $\beta = 5.9$ ($1/a|_\rho = 1.619$ GeV $Z_A = 0.865$) are compared with our $\beta = 5.7$ ($1/a|_\rho = 1.115$ GeV $Z_A = 0.845$) results[1].

The physical values of m_0 (in GeV) extracted from the hairpin analysis (at $\beta = 5.9$) are $.526(14)$, $.533(14)$, $.554(14)$, $.580(14)$, $.606(014)$

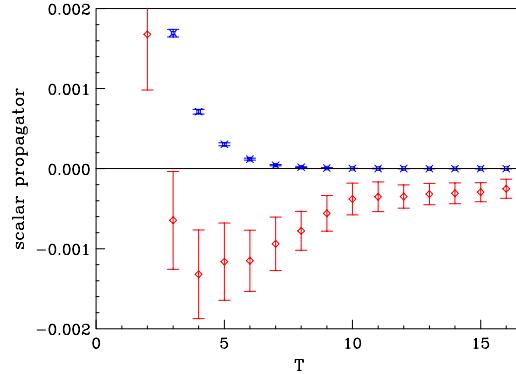


Figure 4. Isovector scalar correlators for $\kappa_q = \kappa_{\bar{q}} = .1397$ (\diamond) and $\kappa_q = \kappa_{\bar{q}} = .1382$ (\times).

Parameter	$\beta = 5.9$	$\beta = 5.7$
$f = f_\pi$	0.091(2)	0.100(2)
$L_5 \times 10^3$	1.55(27)	1.78(35)
$(L_8 - L_5/2) \times 10^3$	0.02(5)	0.14(7)
r_0	1.71(8)	1.99(12)
δ	0.053(13)	0.059(15)

and $.613(17)$ for $m_\pi = .666, .612, .553, .492, .423$ and $.330$ GeV respectively. Extrapolating to $m_\pi = 0$ we obtain $m_0 = .642(15)$. The corresponding value for $\beta = 5.7$ is $m_0 = .548(25)$.

Extracting a_0 and a_1 masses (at κ_c) from scalar and axial vector propagators gives $m_{a_0} = 1.33(5)$ and $1.34(6)$ GeV and $m_{a_1} = 1.27(4)$ and $1.12(8)$ GeV for $\beta = 5.9$ and 5.7 respectively. The value for $m_{a_0} = 1.330(50)$ GeV suggests that the observed $a_0(980)$ resonance is a $K\bar{K}$ “molecule” and not an ordinary $q\bar{q}$ meson.

REFERENCES

1. W. Bardeen, A. Duncan, E. Eichten, and H. Thacker, Phys. Rev. D62 (2000) 114505.
2. J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.
3. W. Bardeen, A. Duncan, E. Eichten, and H. Thacker, Phys. Rev. D57 (1998) 1633; Phys. Rev. D59 (1999) 014507.
4. W. Bardeen, A. Duncan, E. Eichten, N. Ishigur and H. Thacker, Phys. Rev. D 65 (2002) 014509.